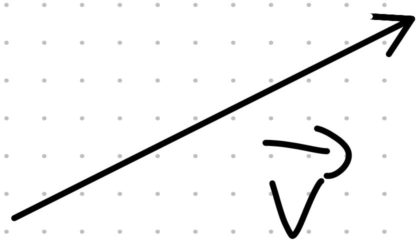
MATH_20C_Lecture_2

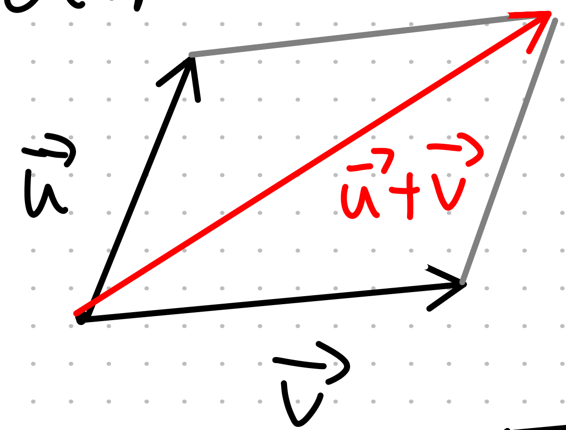
Goal: finish 12.1: 2D vectors

start 12.2: 3D space

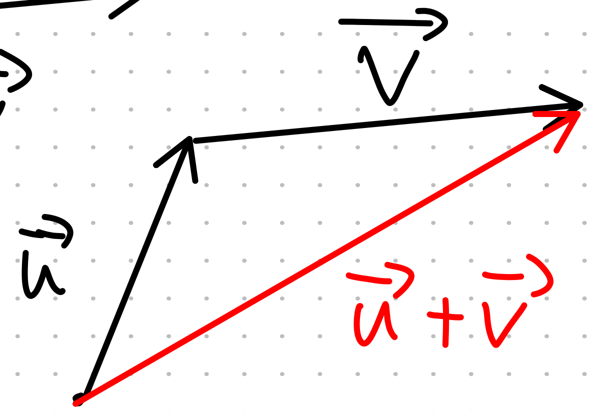
Last time: 2D vector

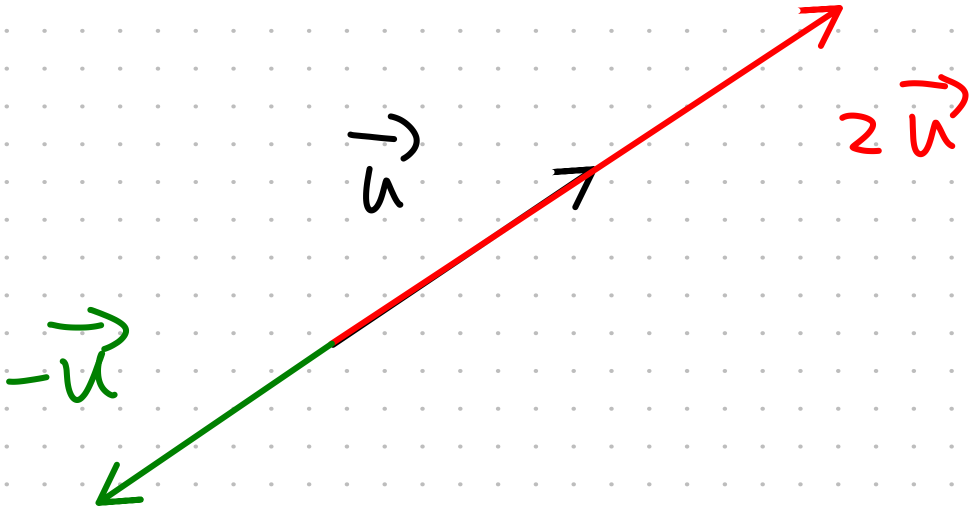
Geometry:



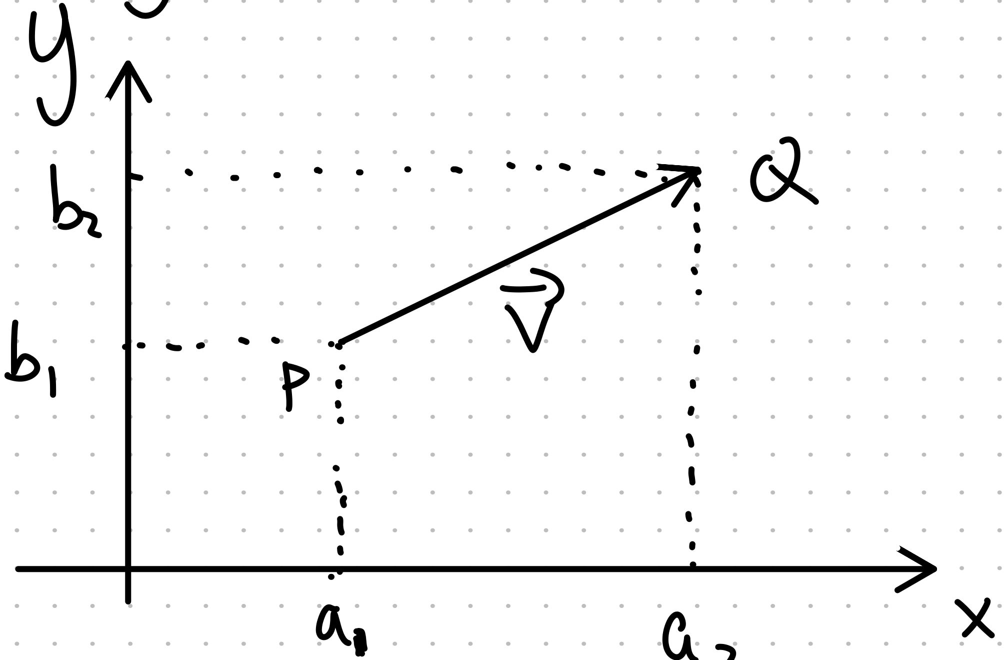


or





Algebraic descriptions:



$$P = (a_1, b_1)$$

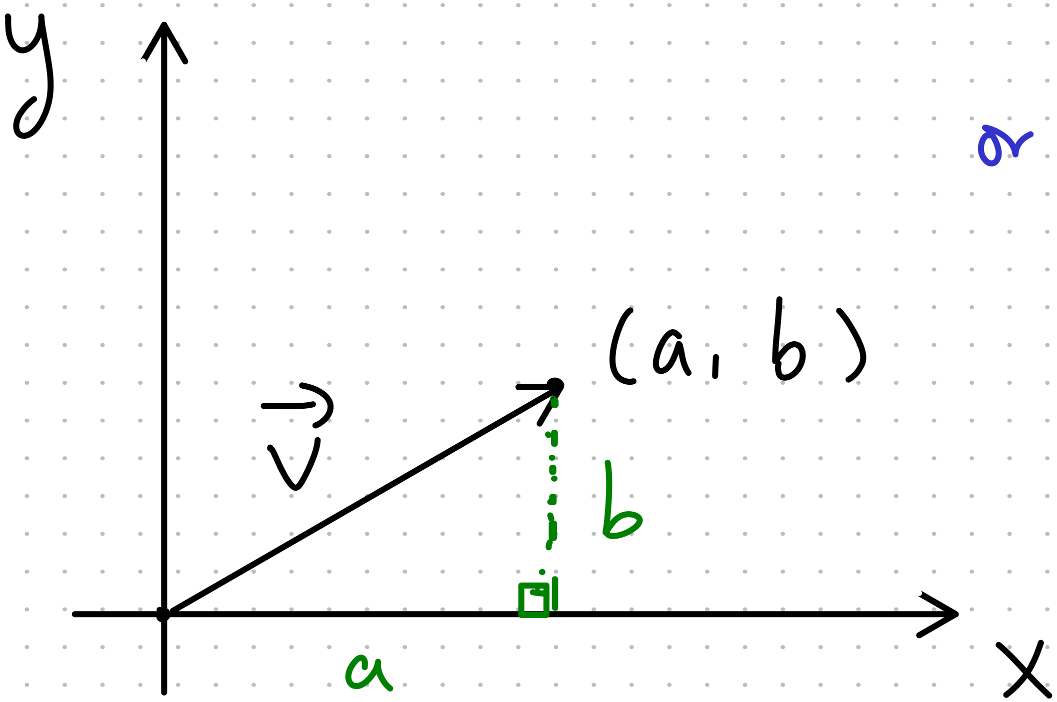
$$Q = (a_2, b_2)$$

$$\vec{v} = \overrightarrow{PQ}$$

$$= \langle a_2 - a_1, b_2 - b_1 \rangle$$

x- coordinate or x- component

y- coordinate or y- component



$$\vec{v} = \langle a, b \rangle$$

$\vec{v} = \langle a, b \rangle$ is the position vector of (a, b)

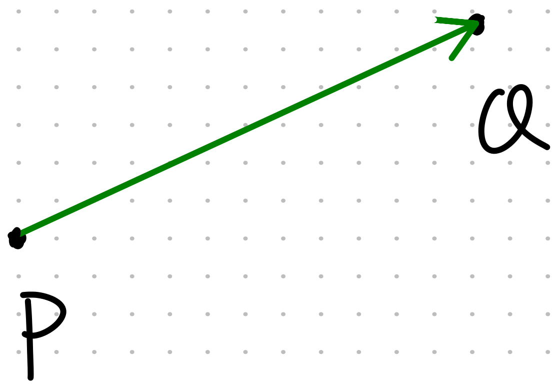
length / magnitude / norm of

$\vec{v} = \langle a, b \rangle$ is defined by

$$\boxed{\|\vec{v}\| = \sqrt{a^2 + b^2}}$$

Ex: $P = (3, 7)$, $Q = (6, 5)$

compute the distance from P to Q



sol: distance $= \|\overrightarrow{PQ}\|$

$$\overrightarrow{PQ} = \langle 6 - 3, 5 - 7 \rangle$$

$$= \langle 3, -2 \rangle$$

$$\|\overrightarrow{PQ}\| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

Exercise: use $\overrightarrow{QP}$ to compute the distance.

<u>vector</u> <u>operations</u>

$\vec{v} = \langle a, b \rangle$, $\vec{w} = \langle c, d \rangle$

(1) $\vec{v} + \vec{w} = \langle a+c, b+d \rangle$

(2) $\vec{v} - \vec{w} = \langle a-c, b-d \rangle$

(3) $\lambda \vec{v} = \langle \lambda a, \lambda b \rangle$ λ scalar

(4) $\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$

$\vec{0} = \langle 0, 0 \rangle$

Ex: $\vec{v} = \langle 1, 4 \rangle$, $\vec{w} = \langle 2, 3 \rangle$

$$3\vec{v} - 2\vec{w} = 3\langle 1,4\rangle - 2\langle 2,3\rangle$$

$$= \langle 3, 12\rangle - \langle 4, 6\rangle$$

$$= \langle 3-4, 12-6\rangle$$

$$= \langle -1, 6\rangle$$

linear combination of $\vec{v}, \vec{w}$

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properties $\vec{u}, \vec{v}, \vec{w}$ vectors

commutative: $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

associative $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$

distributive for scalar mult. $\lambda(\vec{v}+\vec{w}) = \lambda\vec{v}+\lambda\vec{w}$

$\boxed{\text{Def}}$ A <u>linear</u> <u>combination</u> of $\vec{v}$ and $\vec{w}$ is a vector of the form

$$r\vec{v} + s\vec{w}$$ where r, s are scalars.

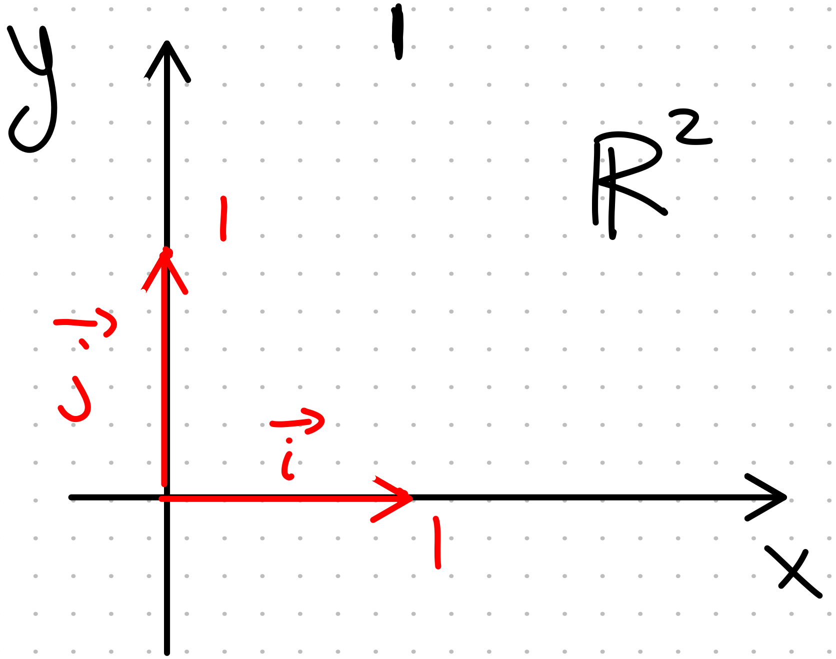
e.g: $2\vec{v}+3\vec{w}$, $5\vec{v}-\vec{w}$

or $\vec{v}$, or $-\vec{v} + 3\vec{w}$

Ex: $\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$

$\mathbf{i}$ $\mathbf{j}$

$\mathbb{R}^2$



$\vec{i}$, $\vec{j}$ are called standard basis vectors.

Fact: Any vector on the 2D plane (i.e. $\mathbb{R}^2$) is a <u>unique</u>

linear comb. of $\vec{i}$ and $\vec{j}$

$$\vec{v} = \boxed{\langle a, b\rangle = a\vec{i} + b\vec{j}}$$

Check: $a\vec{i} + b\vec{j}$

$$= a\langle 1, 0\rangle + b\langle 0, 1\rangle$$

$$= \langle a, 0\rangle + \langle 0, b\rangle$$

$$= \langle a+0, 0+b\rangle$$

$$= \langle a, b\rangle$$

e.g. $\langle 1, 2 \rangle = \vec{i} + 2\vec{j}$

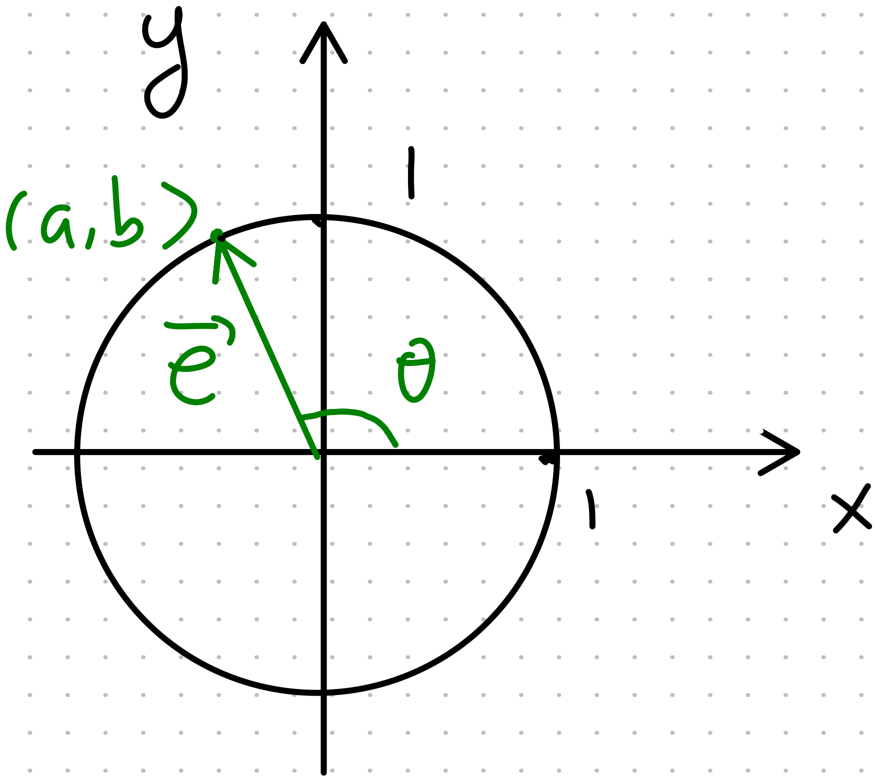
$\boxed{\text{Def}}$ A vector of length 1 is called a <u>unit</u> <u>vector</u>

$$\vec{e} = \langle a, b \rangle$$

$\vec{e}$ is unit $\underbrace{\text{if and only if}}_{\text{iff}}$ $\|\vec{e}\| = 1$

$$\|\vec{e}\| = \sqrt{a^2 + b^2}$$

$\Rightarrow \vec{e}$ is unit iff $a^2+b^2=1$

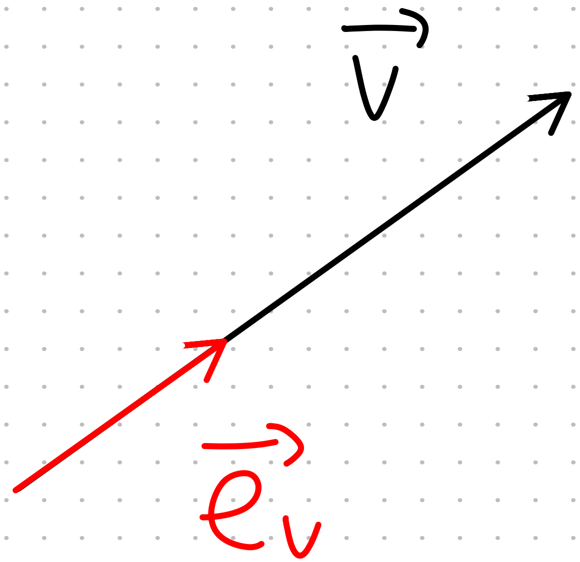


$$\boxed{\begin{aligned}\vec{e} &= \langle a, b\rangle \\ &= \langle \cos\theta, \sin\theta\rangle\end{aligned}}$$

θ is radians

(x degrees $= \dfrac{x}{180}\pi$ radians)

For any $\vec{v} = \langle v_1, v_2 \rangle$, we can obtain a unique unit vector in the same direction.



$$\vec{e}_v = \frac{\vec{v}}{\|\vec{v}\|} \quad \left(= \frac{1}{\|\vec{v}\|} \vec{v} \right)$$

check:

$$\|\vec{e}_v\| = \left\| \frac{1}{\|\vec{v}\|} \vec{v} \right\|$$

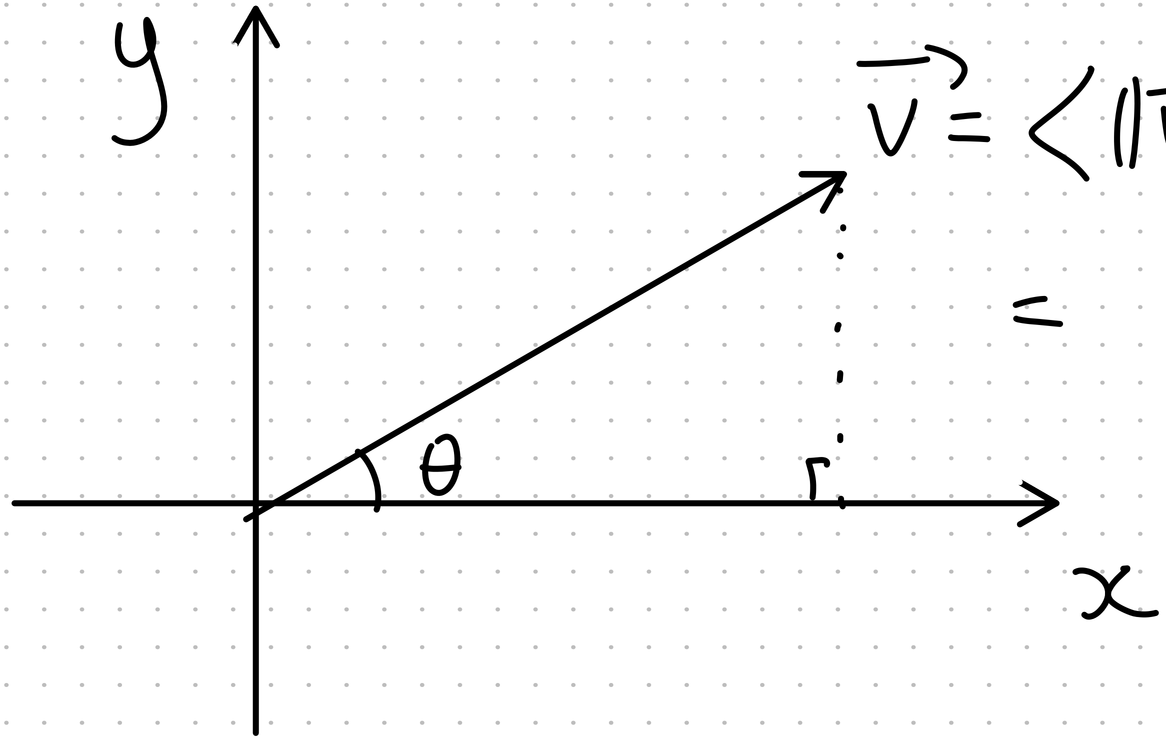
$$= \frac{1}{\cancel{\|\vec{v}\|}} \cancel{\|\vec{v}\|}$$

$$= 1$$

$\Rightarrow$ $\vec{e_v} = \frac{\vec{v}}{\|\vec{v}\|}$ is a unique.

$\Rightarrow$ "polar form of a vector"

$$\vec{v} = \underbrace{\|\vec{v}\|}_{\text{length}} \underbrace{\vec{e_v}}_{\text{direction}}$$



$$\vec{v} = \langle \|\vec{v}\| \cos\theta,\ \|\vec{v}\| \sin\theta \rangle$$

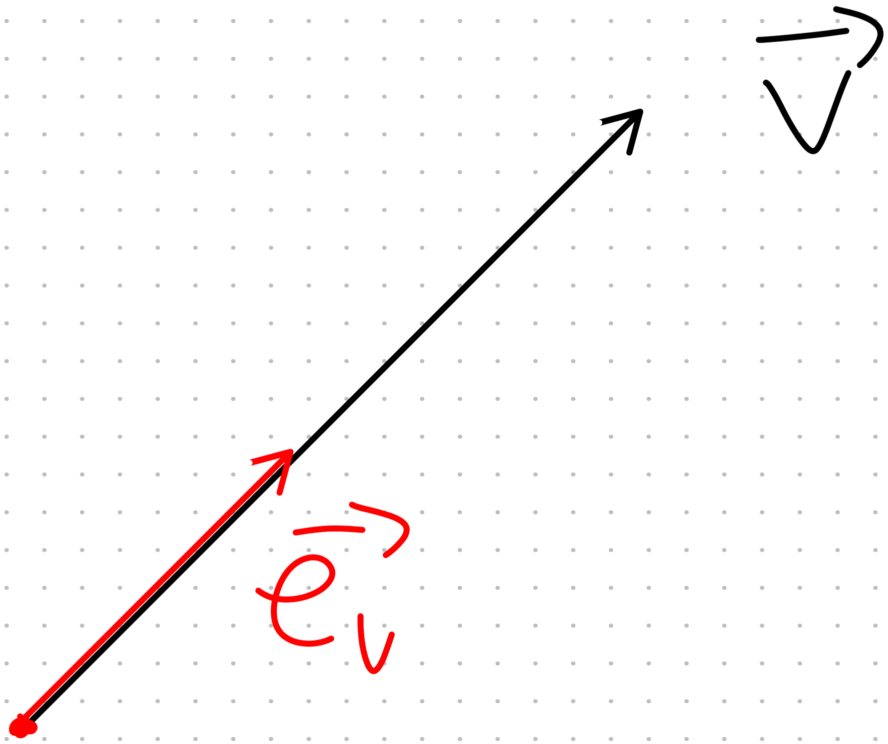
$$= \|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$$

$$= \|\vec{v}\|\ \vec{e}_v$$

Ex: $\vec{v} = \langle 2, 4 \rangle$. Compute $\vec{e}_v$

$$\vec{e}_v = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 2, 4 \rangle}{\sqrt{2^2 + 4^2}} = \frac{\langle 2, 4 \rangle}{\sqrt{20}}$$

"normalize $\vec{v}$"



$$= \left\langle \frac{2}{\sqrt{20}}, \frac{4}{\sqrt{20}} \right\rangle$$

$$\left(= \left\langle \frac{2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}} \right\rangle \right)$$

$$= \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$